

**Indian Statistical Institute, Bangalore**  
**M.Math II year 2018-2019**  
**Semester II : Operator Theory**

Backpaper Exam  
Maximum Marks: 100

Date: 14.06.2019  
Duration: 2 hours

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Note: State the results very clearly that you are using in your answers. All questions carry equal marks.

1. What is the spectrum of the multiplication operator  $M_\phi$  acting on  $L^2[0, 1]$ , for  $\phi$  continuous on  $[0, 1]$ ?
2. Suppose  $T$  is a normal operator on a Hilbert space  $H$ . Show that the spectral radius of  $T$  is  $\|T\|$ .
3. Give an example of a compact linear operator whose spectrum is  $\{0\}$ .
4. Give an example of a Fredholm operator of index  $n$  for  $n \in \mathbb{N}$ .
5. Show that the set of invertible elements in a unital Banach algebra is an open set.